Coded wavefront sensing for video-rate quantitative phase imaging and tomography: validation with digital holographic microscopy: supplemental document

1. APPLICATION OF THE FOURIER SLICE THEOREM

As the HEK cell is immersed in PBS (RI $= 1.334$), the scattering potential of the HEK293 cell is small, corresponding to $\max(\Delta\eta) \approx 0.06$, based on commonly found intracellular matter such as the nucleus or the nucleolus [1]. The weak scattering implies the applicability of the Fourier diffraction theorem (FDT) for tomography [2]. With FDT, object frequencies (in wave vector-space) up to $\sqrt{2}k_m = 18.52\,\mathrm{rad}\,\mu\mathrm{m}^{-1}$ can be retrieved, where $k_m = \frac{2\pi\eta_m}{\lambda_0}$, $\lambda_0 = 640\,\mathrm{nm}$, and $\eta_m = 1.334$ is the RI of PBS. The spectrum of each measurement maps to a hemispherical surface with radius k_m in the spectrum of the specimen.

In this paper, our goal is to demonstrate that while the integration of Coded WFS with standard microscopes is simple (as evidenced by Fig. 1 **a** in the main manuscript), it enables both quantitative phase imaging of dynamic specimens and tomography. Therefore, we apply a smoothing Gaussian filter with $\sigma = 3.5$ as it immediately simplifies the tomography problem in Sect. 3.4 (of the main manuscript) by (i) mitigating the specimen jitter observed between frames and (ii) limiting the frequency coverage of the OPD projections of the 3D RI distribution. The latter provides tolerance against errors in the approximated pose and enables the use of the simpler Fourier slice theorem (FST). We provide further details about the applicability of FST here.

Fig. S1 shows the phase $\phi(x, y)$ [in rad] of a single HEK293 cell measurement retrieved by Coded WFS and its spectrum (in log-scale) in the object space. The radius of the dashed green circle enclosing the spatial frequencies collected by the NA of the objective ($\mathrm{NA_{obj}} = 1.15$) is given by $\pi\eta_m \times \frac{\mathrm{NA_{obj}}}{0.61\lambda_0} = 12.34\,\mathrm{rad}\,\mu\mathrm{m}^{-1}$.

After convolving the retrieved phase with a Gaussian filter, the resulting spectrum as shown in Fig. S2 (left) is enclosed by the dashed circle with radius $5.5\,\mathrm{rad}\,\mu\mathrm{m}^{-1}$ in the case of Coded WFS and $6.5\,\mathrm{rad}\,\mu\mathrm{m}^{-1}$ for DHM. In Fig. S2 (right), we draw the 2D semicircular arcs with radius k_m centered $||k_m||$ from the center of the spectrum to display the angular frequency support provided by FDT. When the spectrum of the measurements is mapped to the spectrum of the specimen Fig. S2, the frequency coverage is limited to the dashed red and blue circles for Coded WFS and DHM, respectively. Within this region, the curvature of the hemispherical surfaces is small, and if approximated with planes instead, it would be equivalent to the Fourier slice theorem. Note that errors in this approximation are smaller for lower frequencies, which have larger coefficients. Henceforth, we use the FST algorithm to retrieve a low-pass filtered 3D RI distribution of the object shown in Fig. 4 in the main manuscript.

REFERENCES

1. P. Y. Liu, L. K. Chin, W. Ser, *et al.*, "Cell refractive index for cell biology and disease diagnosis: Past, present and future," Lab on a Chip **16**, 634–644 (2016).
2. A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging* (SIAM, 2001).

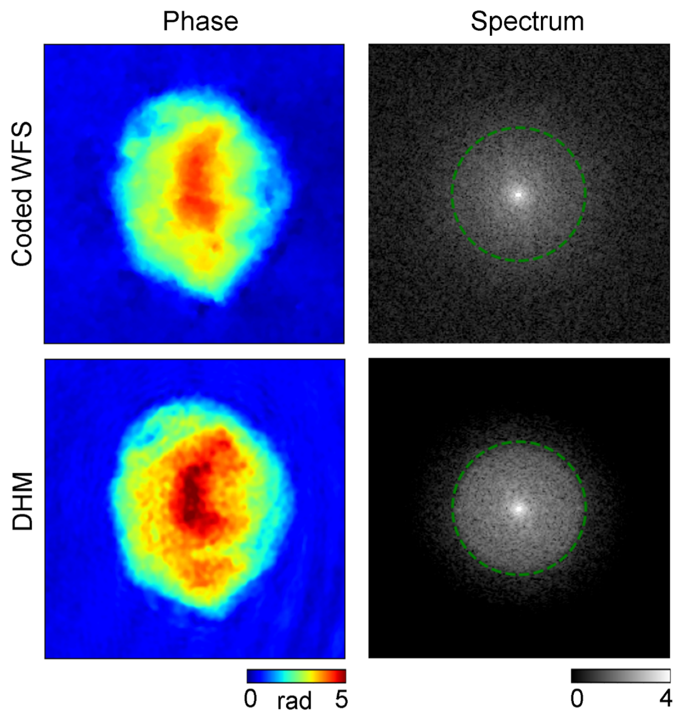

Fig. S1. Frequency coverage of the optical system. Phases [rad] retrieved using Coded WFS and DHM (left) and the magnitude of their spectrum [log −space] (right). The spatial frequencies are limited by the NA of the objective drawn with dashed green circles.

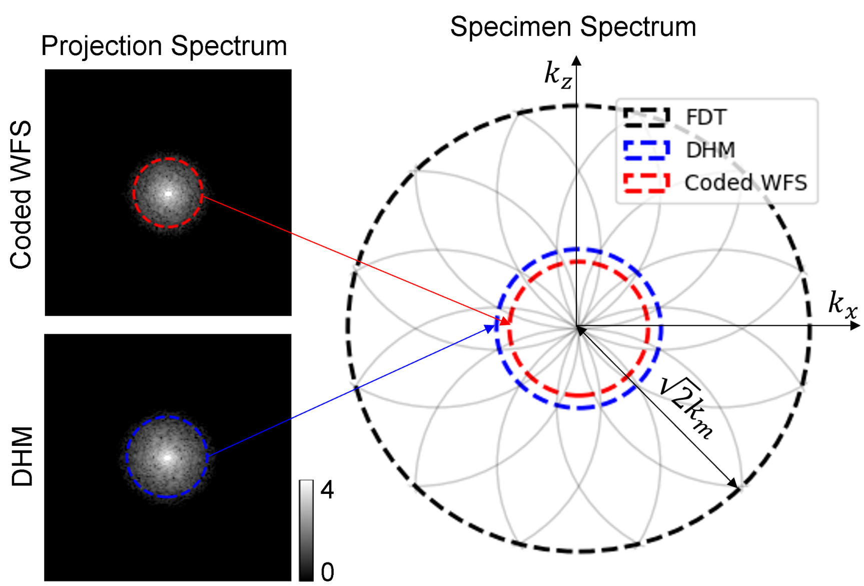

Fig. S2. Mapping projection spectrum to specimen spectrum. The magnitude of the spectrum [log −space] of the spatially smoothed phases (left) are mapped to the spectrum of the specimen (right). On the right, the dashed black circle encloses the frequency coverage provided by FDT, whereas the dashed red and blue circles enclose the maximum frequency coverage provided by the filtered phases using Coded WFS and DHM, respectively.